

# Package ‘BBSSL’

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**Title** Bayesian Bootstrap Spike-and-Slab LASSO

**Version** 0.1.0

**Description** Posterior sampling for Spike-and-Slab LASSO prior in linear models from Nie and Rockova <arXiv:2011.14279>.

**License** GPL-3

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BB\_SSL

*BB-SSL***Description**

This function runs BB-SSL, WBB with fixed prior weight, and WBB with random prior weight. It solves the optimization by calling function `SSLASSO_2`, a variant of the function `SSLASSO` in CRAN package 'SSLASSO': in the version used, we do NOT standardize the design matrix and allow inputting initial values of beta's.

**Usage**

```
BB_SSL(y, X, method = 3, lambda, NSample, a, b, maxiter=500, eps = 1e-3, burn.in = FALSE,
length.out = 50, discard = FALSE, alpha = 3, sigma = 1, initial.beta,
penalty = "adaptive", theta=0.5)
```

**Arguments**

<code>y</code>	A vector of continuous responses (n x 1).
<code>X</code>	The design matrix (n x p), without an intercept.
<code>method</code>	A number between c(1,2,3) to specify which method to run, method = 1 is fixed WBB, method = 2 is random WBB, method = 3 is BB-SSL.
<code>lambda</code>	A two-dim vector = c(lambda0, lambda1).
<code>NSample</code>	An integer which specifies the number of samples to be generated.
<code>a, b</code>	Parameters of the prior.
<code>maxiter</code>	An integer which specifies the maximum number of iterations for <code>SSLASSO_2</code> (default maxiter= 500).
<code>eps</code>	Convergence criterion when running <code>SSLASSO_2</code> : converged when difference in regression coefficients is less than eps (default eps = 0.001).
<code>burn.in</code>	A boolean to specify whether to use annealing on a sequence of lambda0's (default burn.in = FALSE).
<code>length.out</code>	An integer to specify the length of sequence of lambda0's used in annealing. This value is not used when burn.in = FALSE. Default is 50.
<code>discard</code>	A boolean to specify whether to discard unconverged sample points.
<code>alpha</code>	The parameter for generating weights in BB-SSL, which follows n x Dirichlet(alpha,...,alpha). Default is 3.
<code>sigma</code>	Noise standard deviation.
<code>initial.beta</code>	A vector of initial values of beta to used when solving <code>SSLASSO_2</code> (n x 1).
<code>penalty</code>	The penalty (prior) to be applied to the model. Either "separable" (with a fixed theta) or "adaptive" (with a random theta, where $\theta \sim B(a,p)$ ). The default is "adaptive".
<code>theta</code>	Prior mixing proportion. For "separable" penalty, this value is fixed. For "adaptive" penalty, this value is used as a starting value. Default is 0.5.

**Value**

A list of matrices, including matrix beta (NSample x p) and matrix gamma (NSample x p).

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**References**

Nie, L., & Ročková, V. (2020). Bayesian Bootstrap Spike-and-Slab LASSO. arXiv:2011.14279.

Newton, M. A., Polson, N. G., and Xu, J. (2020). Weighted Bayesian bootstrap for scalable posterior distributions. Canadian Journal of Statistics (In Press).

**Examples**

```
# ----- Generate Data -----
n = 50; p = 12;
truth.beta = c(1.3, 1.3, 1.3, 1.3);
truth.sigma = 1
data = Generate_data(truth.beta, p, n, truth.sigma = 1, rho = 0.6,"block",4)
y = data$y; X = data$X; beta = data$beta

# ----- set parameters -----
lambda0 = 7; lambda1 = 0.15; lambda = c(lambda0, lambda1)
a = 1; b = p #beta prior for theta

#----- BB-SSL -----
# this is for demonstration of usage only
# in practice, you may want to use more iterations!
BB.SSL.result = BB_SSL(y, X, method = 3, lambda = c(lambda0, lambda1), NSample = 100, a, b,
maxiter = 500, length.out = 50, burn.in = FALSE, discard = TRUE, alpha=1,
initial.beta = rep(0,p))

# Alternatively, you can use SSLASSO_2 solution to get an initial value of beta's
result = SSLASSO_2(X, y, penalty = "adaptive", variance = "fixed", sigma = truth.sigma,
lambda1 = lambda1, lambda0 = seq(lambda1, lambda0, length.out = 50),
a = a, b = b,
max.iter = 500, initial.beta = rep(0,p))

fixed.WBB.result = BB_SSL(y, X, method = 1, lambda = c(lambda0, lambda1), NSample = 100,
a, b, maxiter = 500, length.out = 50, burn.in = FALSE,
discard = TRUE, initial.beta = result$beta[,50])

random.WBB.result = BB_SSL(y, X, method = 2, lambda = c(lambda0, lambda1), NSample = 100,
a, b, maxiter = 500, length.out = 50, burn.in = FALSE,
discard = TRUE, initial.beta = result$beta[,50])

BB.SSL.result = BB_SSL(y, X, method = 3, lambda = c(lambda0, lambda1), NSample = 100, a,
b, maxiter = 500, length.out = 50, burn.in = FALSE, discard = TRUE,
```

```
alpha=1, initial.beta = result$beta[,50])
```

---

Generate\_data

*Generate\_data*

---

### Description

This function generates data (X,y) with specified correlation and noise standard deviation.

### Usage

```
Generate_data(truth.beta, p, n, truth.sigma, rho, correlation = c("block", "all"),
  NumOfBlock)
```

### Arguments

truth.beta	A vector of active beta's (s x 1, with s being the number of active coordinates).
p	The number of covariates.
n	The number of observations.
truth.sigma	Noise standard deviation.
rho	Correlation Coefficient.
correlation	Correlation structure. Correlation = "block" means predictors are grouped into equi-size blocks where each block contains one active predictor, and the within-block correlation coefficient is rho; predictors in different blocks are mutually independent. Correlation = "all" means all predictors are equi-correlated with coefficient rho.
NumOfBlock	Number of blocks, used only when correlation = 'block'.

### Value

A list, including vector 'y' (n x 1), matrix 'X' (n x p), vector 'beta' (p x 1).

---

Gibbs

*Gibbs*

---

### Description

This function runs SSVS for linear regression with Spike-and-Slab LASSO prior. By default, this function uses the speed-up trick in Bhattacharya et al. (2016) when  $p > n$ .

### Usage

```
Gibbs(y, X, a, b, lambda, maxiter, burn.in, initial.beta = NULL, sigma = 1)
```

**Arguments**

<code>y</code>	A vector of continuous responses ( $n \times 1$ ).
<code>X</code>	The design matrix ( $n \times p$ ), without an intercept.
<code>a, b</code>	Parameters of the prior.
<code>lambda</code>	A two-dim vector = $c(\text{lambda0}, \text{lambda1})$ .
<code>maxiter</code>	An integer which specifies the maximum number of iterations for MCMC.
<code>burn.in</code>	An integer which specifies the maximum number of burn-in iterations for MCMC.
<code>initial.beta</code>	A vector of initial values of beta to used. If set to NULL, the LASSO solution with 10-fold cross validation is used. Default is NULL.
<code>sigma</code>	Noise standard deviation. Default is 1.

**Value**

A list, including matrix 'beta' ( $(\text{maxiter}-\text{burn.in}) \times p$ ), matrix 'tau2' ( $(\text{maxiter}-\text{burn.in}) \times p$ ), matrix 'gamma' ( $(\text{maxiter}-\text{burn.in}) \times p$ ), vector 'theta' ( $(\text{maxiter}-\text{burn.in}) \times 1$ ).

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**References**

- Nie, L., & Ročková, V. (2020). Bayesian Bootstrap Spike-and-Slab LASSO. arXiv:2011.14279.
- Bhattacharya, A., Chakraborty, A., & Mallick, B. K. (2016). Fast sampling with Gaussian scale mixture priors in high-dimensional regression. *Biometrika*, 103(4):985.

**Examples**

```
n = 50; p = 12;
truth.beta = c(1.3, 1.3, 1.3, 1.3);
truth.sigma = 1
data = Generate_data(truth.beta, p, n, truth.sigma = 1, rho = 0.6, "block", 4)
y = data$y; X = data$X; beta = data$beta

# ----- set parameters -----
lambda0 = 7; lambda1 = 0.15; lambda = c(lambda0, lambda1)
a = 1; b = p #beta prior for theta

# this is for demonstration of usage only
# in practice, you may want to use more iterations!
MCchain1 = Gibbs(y, X, a, b, lambda, maxiter = 1000, burn.in = 100)
```

Gibbs2

*Gibbs2***Description**

This function runs one-site Gibbs sampler for linear regression with Spike-and-Slab LASSO prior.

**Usage**

```
Gibbs2(y, X, a, b, lambda, maxiter, burn.in, initial.beta = NULL, sigma = 1)
```

**Arguments**

<code>y</code>	A vector of continuous responses (n x 1).
<code>X</code>	The design matrix (n x p), without an intercept.
<code>a, b</code>	Parameters of the prior.
<code>lambda</code>	A two-dim vector = c(lambda0, lambda1).
<code>maxiter</code>	An integer which specifies the maximum number of iterations for MCMC.
<code>burn.in</code>	An integer which specifies the number of burn-in iterations for MCMC.
<code>initial.beta</code>	A vector of initial values of beta to used. If set to NULL, the LASSO solution with 10-fold cross validation is used. Default is NULL.
<code>sigma</code>	Noise standard deviation. Default is 1.

**Value**

A list, including matrix beta ((maxiter-burn.in) x p) and matrix gamma (maxiter-burn.in) x p, vector theta ((maxiter-burn.in) x 1)

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**References**

Nie, L., & Ročková, V. (2020). Bayesian Bootstrap Spike-and-Slab LASSO. arXiv:2011.14279.

**Examples**

```
n = 50; p = 12;
truth.beta = c(1.3, 1.3, 1.3, 1.3);
truth.sigma = 1
data = Generate_data(truth.beta, p, n, truth.sigma = 1, rho = 0.6, "block", 4)
y = data$y; X = data$X; beta = data$beta

# ----- set parameters -----
lambda0 = 7; lambda1 = 0.15; lambda = c(lambda0, lambda1)
```

```

a = 1; b = p #beta prior for theta

# this is for demonstration of usage only
# in practice, you may want to use more iterations!
MCchain2 = Gibbs2(y, X, a, b, lambda, maxiter = 1000, burn.in = 100)

```

---

plot.SSLASSO	<i>Plot coefficients from a "SSLASSO" object</i>
--------------	--

---

### Description

Produces a plot of the coefficient paths for a fitted "SSLASSO" object.

### Usage

```

## S3 method for class 'SSLASSO'
plot(x, ...)

```

### Arguments

x	Fitted "SSLASSO" model.
...	Other graphical parameters to plot.

### Value

A plot of the coefficient paths for a fitted "SSLASSO" object.

### References

Rockova, V. and George, E.I. (2018) The Spike-and-Slab LASSO. Journal of the American Statistical Association.

### See Also

[SSLASSO\\_2](#)

### Examples

```

## Linear regression, where p>n

n = 100; p = 1000;
truth.beta = c(2, 3, -3, 4); # high-dimensional case
truth.sigma = 1
data = Generate_data(truth.beta, p, n, truth.sigma = 1, rho = 0.6,"all", 4)
y = data$y; X = data$X; beta = data$beta

# ----- set parameters -----
lambda0 = 50; lambda1 = 0.05; lambda = c(lambda0, lambda1)

```

```

a = 1; b = p #beta prior for theta

# Separable penalty with fixed theta

result<-SSLASSO_2(X, y,penalty="separable", variance = "fixed",
lambda1 = lambda1, lambda0 = seq(from=lambda1,to=lambda0,length.out=50),
theta = 4/p,initial.beta = rep(0,p))

plot(result)

```

---

SSLASSO\_2

*The Spike-and-Slab LASSO (for BB-SSL).*


---

### Description

Spike-and-Slab LASSO is a spike-and-slab refinement of the LASSO procedure, using a mixture of Laplace priors indexed by  $\lambda_{0}$  (spike) and  $\lambda_{1}$  (slab).

The SSLASSO procedure fits coefficients paths for Spike-and-Slab LASSO-penalized linear regression models over a grid of values for the regularization parameter  $\lambda_{0}$ . The code has been adapted from the SSLASSO package (Rockova, V. and Moran, G. (2019). Package ‘SSLASSO’.) such that now it does NOT normalize each column and allows specifying initialization value).

### Usage

```

SSLASSO_2(X, y, initial.beta, penalty = c("adaptive", "separable"),
variance = c("fixed", "unknown"), lambda1, lambda0, nlambda = 100,
theta = 0.5, sigma = 1, a = 1, b, eps = 0.001, max.iter = 500,
counter = 10, warn = FALSE)

```

### Arguments

<code>X</code>	The design matrix ( $n \times p$ ), without an intercept. SSLASSO standardizes the data by default.
<code>y</code>	Vector of continuous responses ( $n \times 1$ ). The responses will be centered by default.
<code>initial.beta</code>	Initial value for beta when searching for the solution.
<code>penalty</code>	The penalty to be applied to the model. Either "separable" (with a fixed theta) or "adaptive" (with a random theta, where $\theta \sim B(a,p)$ ). The default is "adaptive".
<code>variance</code>	Whether the error variance is also estimated. Either "fixed" (with a fixed sigma) or "unknown" (with a random sigma, where $p(\sigma) \sim 1/\sigma$ ). The default is "fixed".
<code>lambda1</code>	Slab variance parameter. Needs to be less than $\lambda_{0}$ . The default is $\lambda_{0} = 1$ .



<code>lambda0</code>	Spike penalty parameters ( $L \times 1$ ). Either a numeric value for a single run ( $L=1$ ) or a sequence of increasing values for dynamic posterior exploration. The default is <code>lambda0 = seq(1, nrow(X), length.out = 100)</code> .
<code>nlambda</code>	The number of <code>lambda0</code> values. Default is 100.
<code>theta</code>	Prior mixing proportion. For "separable" penalty, this value is fixed. For "adaptive" penalty, this value is used as a starting value.
<code>sigma</code>	Error variance. For "fixed" variance, this value is fixed. For "unknown" variance, this value is used as a starting value.
<code>a</code>	Hyperparameter of the beta prior $B(a, b)$ for the adaptive penalty (default $a = 1$ ).
<code>b</code>	Hyperparameter of the beta prior $B(a, b)$ for the adaptive penalty (default $b = \text{ncol}(X)$ ).
<code>eps</code>	Convergence criterion: converged when difference in regression coefficients is less than <code>eps</code> (default <code>eps = 0.001</code> ).
<code>max.iter</code>	Maximum number of iterations. Default is 500.
<code>counter</code>	Applicable only for the adaptive penalty. Determines how often the parameter <code>theta</code> is updated throughout the cycles of coordinate ascent. Default is 10.
<code>warn</code>	TRUE if warnings should be printed; FALSE by default

### Details

The sequence of models indexed by the regularization parameter `lambda0` is fitted using a coordinate descent algorithm. The algorithm uses screening rules for discarding irrelevant predictors along the lines of Breheny (2011).

### Value

An object with S3 class "SSLASSO" containing:

<code>beta</code>	The fitted matrix of coefficients ( $p \times L$ ). The number of rows is equal to the number of coefficients $p$ , and the number of columns is equal to $L$ (the length of <code>lambda0</code> ).
<code>intercept</code>	A vector of length $L$ containing the intercept for each value of <code>lambda0</code> . The intercept is <code>intercept = mean(y) - crossprod(XX, beta)</code> , where <code>XX</code> is the centered design matrix.
<code>iter</code>	A vector of length $L$ containing the number of iterations until convergence at each value of <code>lambda0</code> .
<code>lambda0</code>	The sequence of regularization parameter values in the path.
<code>penalty</code>	Same as above.
<code>thetas</code>	A vector of length $L$ containing the hyper-parameter values <code>theta</code> (the same as <code>theta</code> for "separable" penalty).
<code>sigmas</code>	A vector of length $L$ containing the values <code>sigma</code> (the same as the initial <code>sigma</code> for "known" variance).
<code>select</code>	A ( $p \times L$ ) binary matrix indicating which variables were selected along the solution path.
<code>model</code>	A single model chosen after the stabilization of the regularization path.

## References

Rockova, V. and George, E.I. (2018) The Spike-and-Slab LASSO. *Journal of the American Statistical Association*.

Moran, G., Rockova, V. and George, E.I. (2018) On variance estimation for Bayesian variable selection. <<https://arxiv.org/abs/1801.03019>>.

Nie, L., & Ročková, V. (2020). Bayesian Bootstrap Spike-and-Slab LASSO. arXiv:2011.14279.

## Examples

```
## Linear regression, where p > n

p <- 1000
n <- 100

X <- matrix(rnorm(n*p), nrow = n, ncol = p)
beta <- c(1, 2, 3, rep(0, p-3))
y = X[,1] * beta[1] + X[,2] * beta[2] + X[,3] * beta[3] + rnorm(n)

# Oracle SSLASSO with known variance

result1 <- SSLASSO_2(X, y, penalty = "separable", theta = 3/p, initial.beta = rep(0,p))
plot(result1)

# Adaptive SSLASSO with known variance

result2 <- SSLASSO_2(X, y, initial.beta = rep(0,p))
plot(result2)
```

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